Section 4.
Stellar evolution

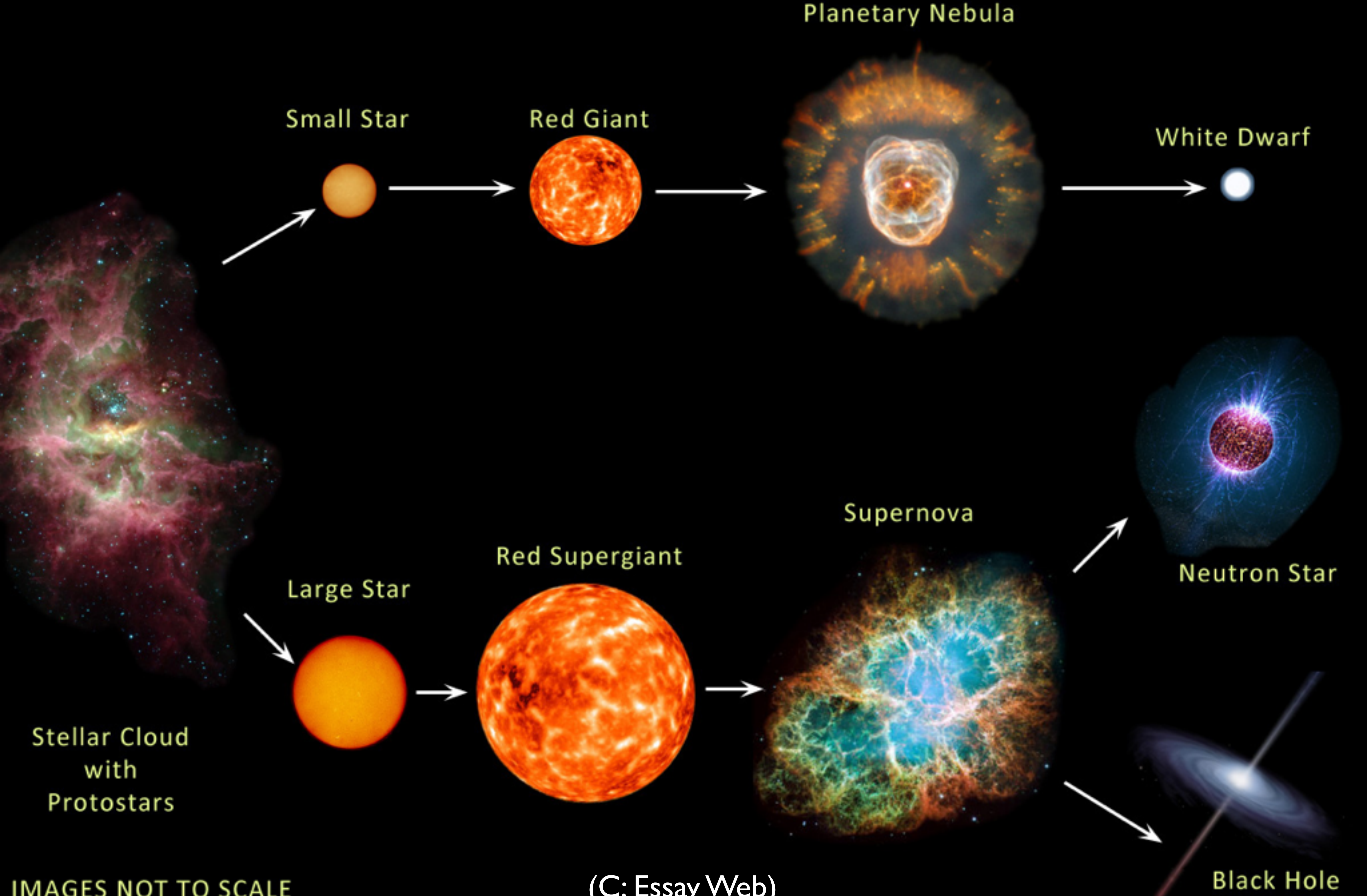
4.1 Virial theorem

4.2 Evolution of density and temperature

Let's understand these questions with the words of physics

- Why are stars so luminous?
- Why do stars show L ~ M^4?
- Why do stars evolve?
- Why does the destiny of stars depend on the mass?
- Why do some stars explode?
- Why don't normal star explode?
- Why does stellar core collapses?
- Why is the energy of supernova so huge?
- …

Stellar life



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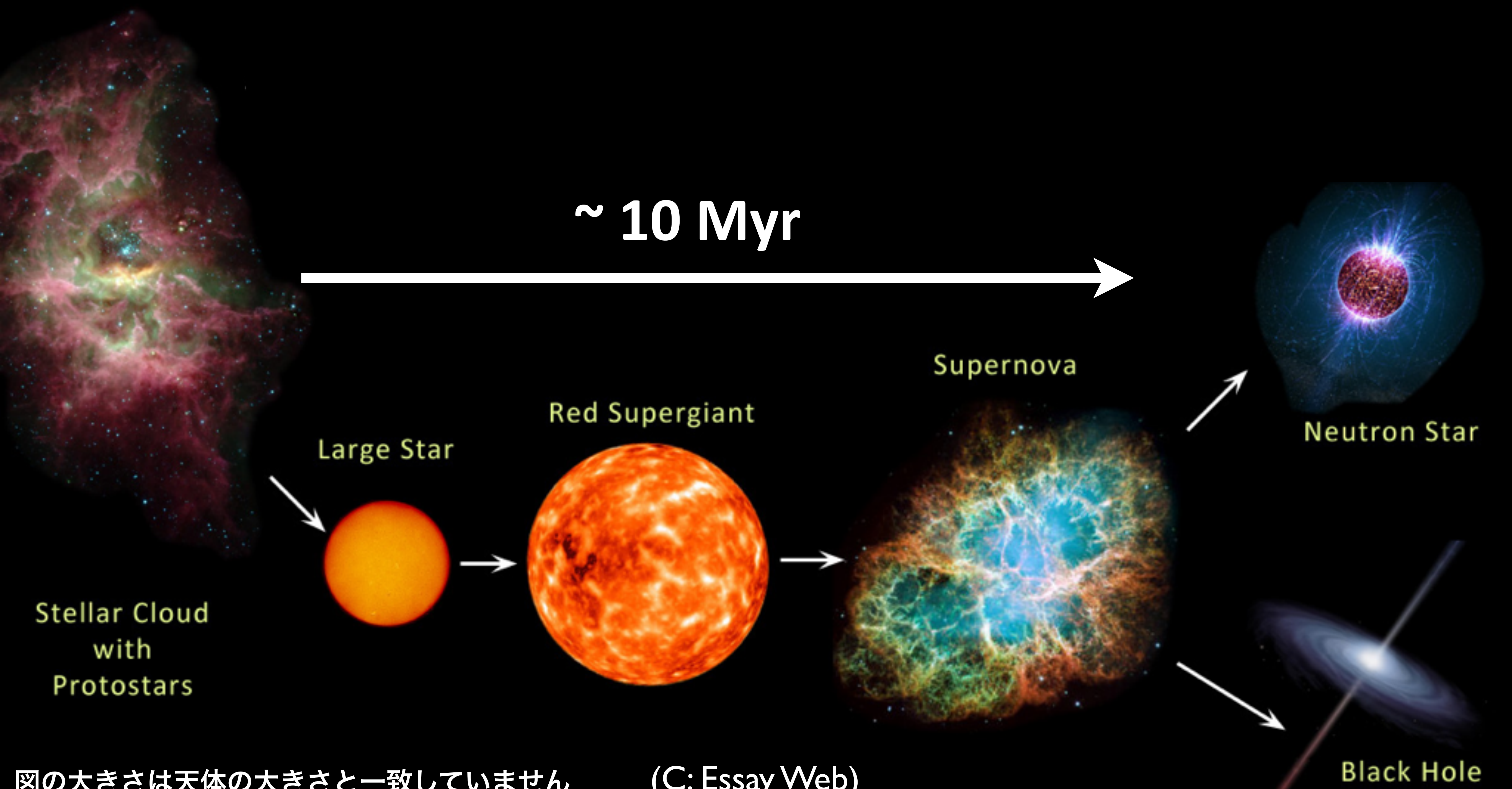
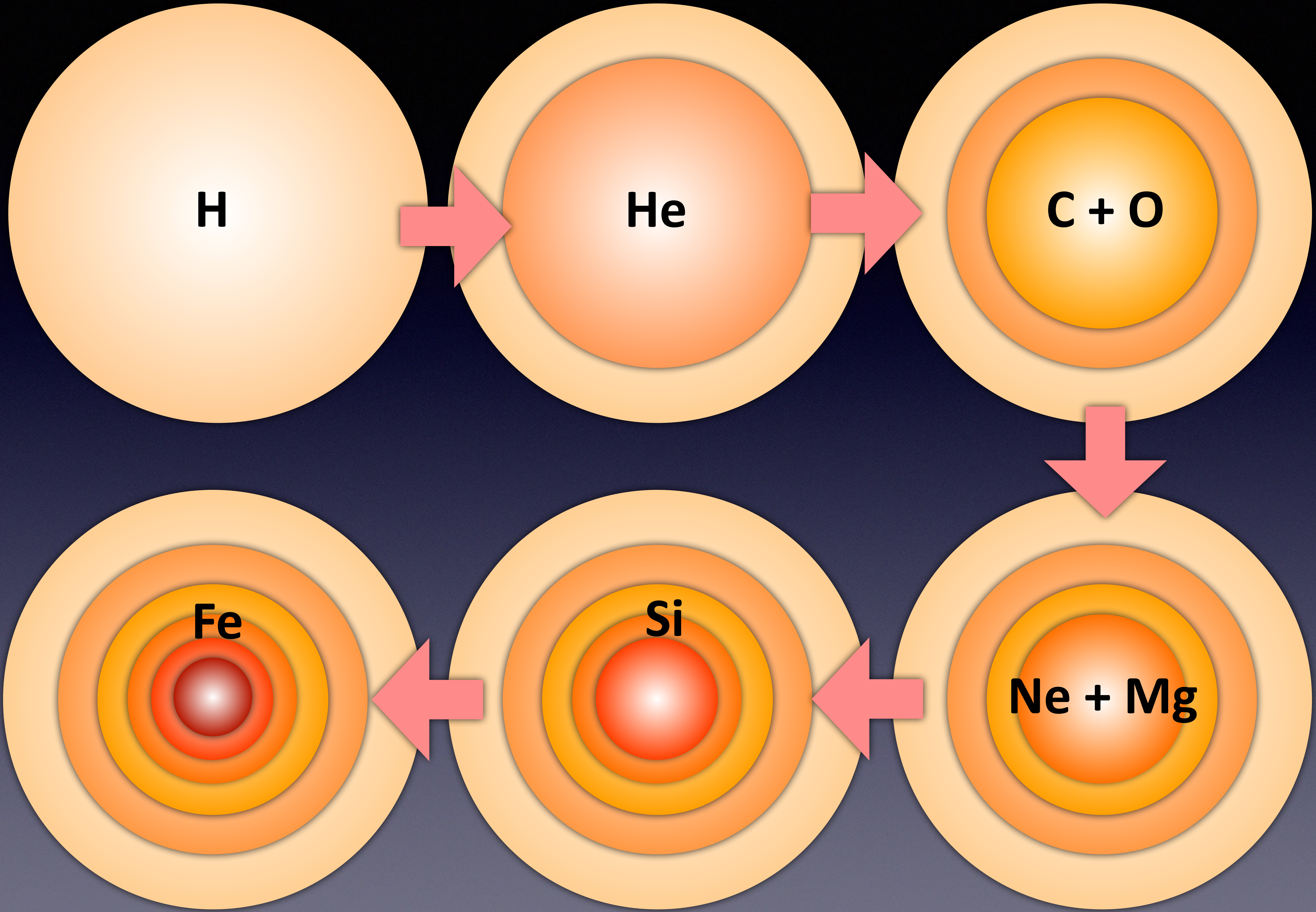
1. Massive stars

M > 10 Msun



図の大きさは天体の大きさと一致していません (C: Essay Web)



Images are not to scale

Why do stars evolve??

"Evolution" = Changes in the state with time

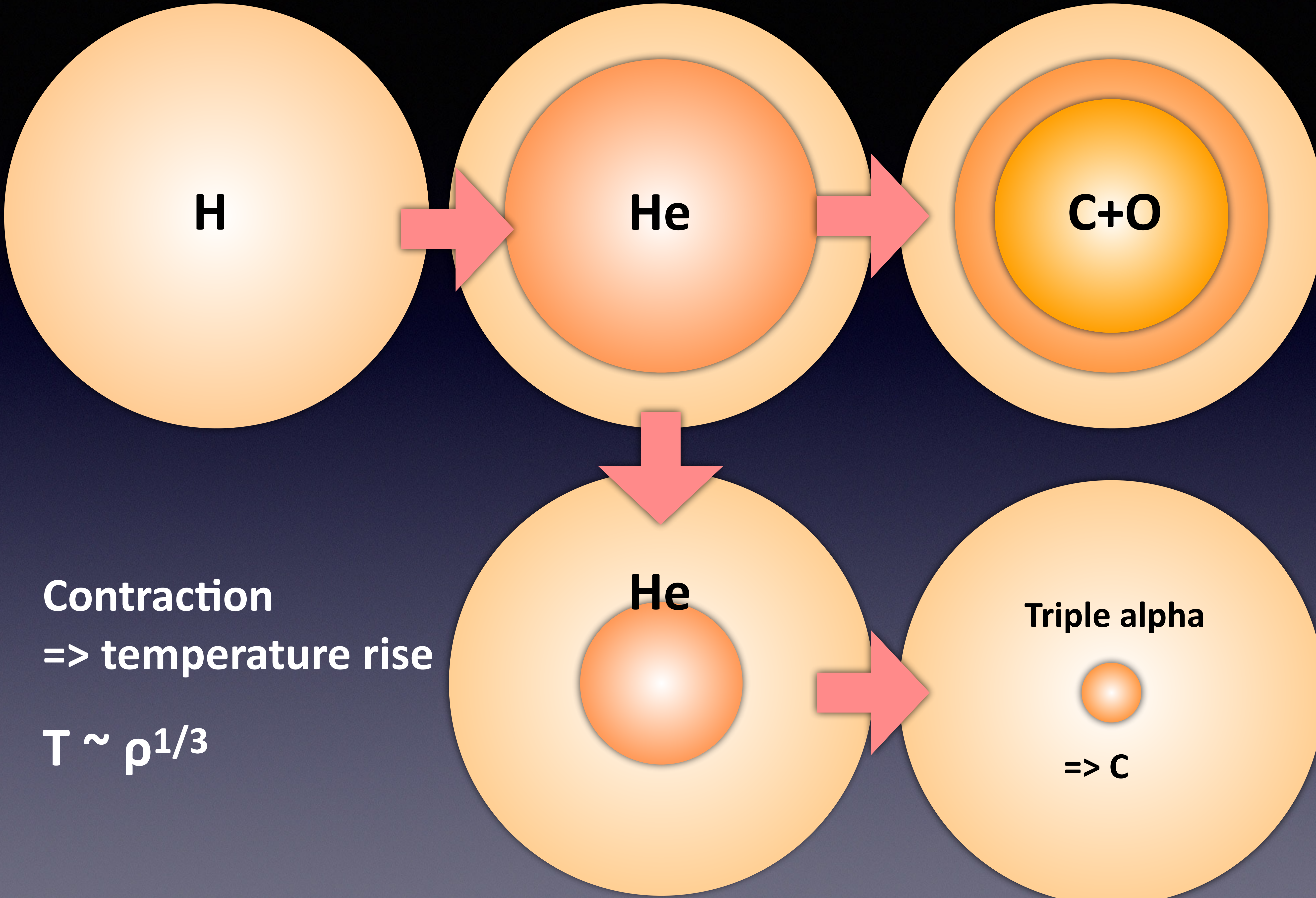
What happens when there is no more fuel for nuclear burning

E_{tot}: Total energy
Ω: Gravitational energy
U: Internal energy

$$U = -\frac{1}{2}\Omega$$

$$E_{\rm tot} = U + \Omega = \frac{1}{2}\Omega = -U$$

No nuclear burning
- Total energy decreases
- Contraction (gravitational energy decreases)
- Temperature rises



Contraction
=> temperature rise

$T \sim \rho^{1/3}$

Images are not to scale

Heated iron

stars

Gets colder

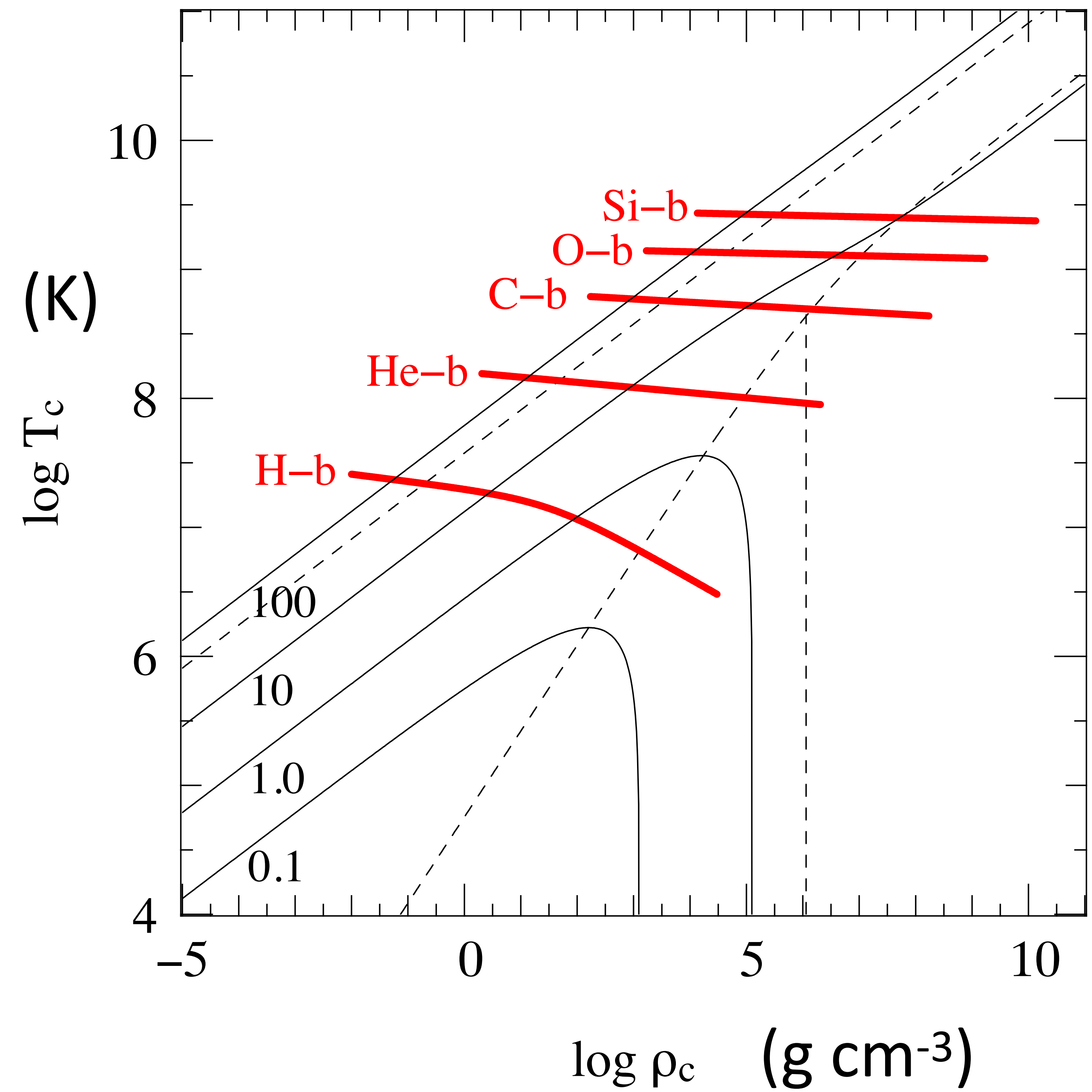
Gets hotter

Condition of H-burning

Fusion reactor

~10^8 K

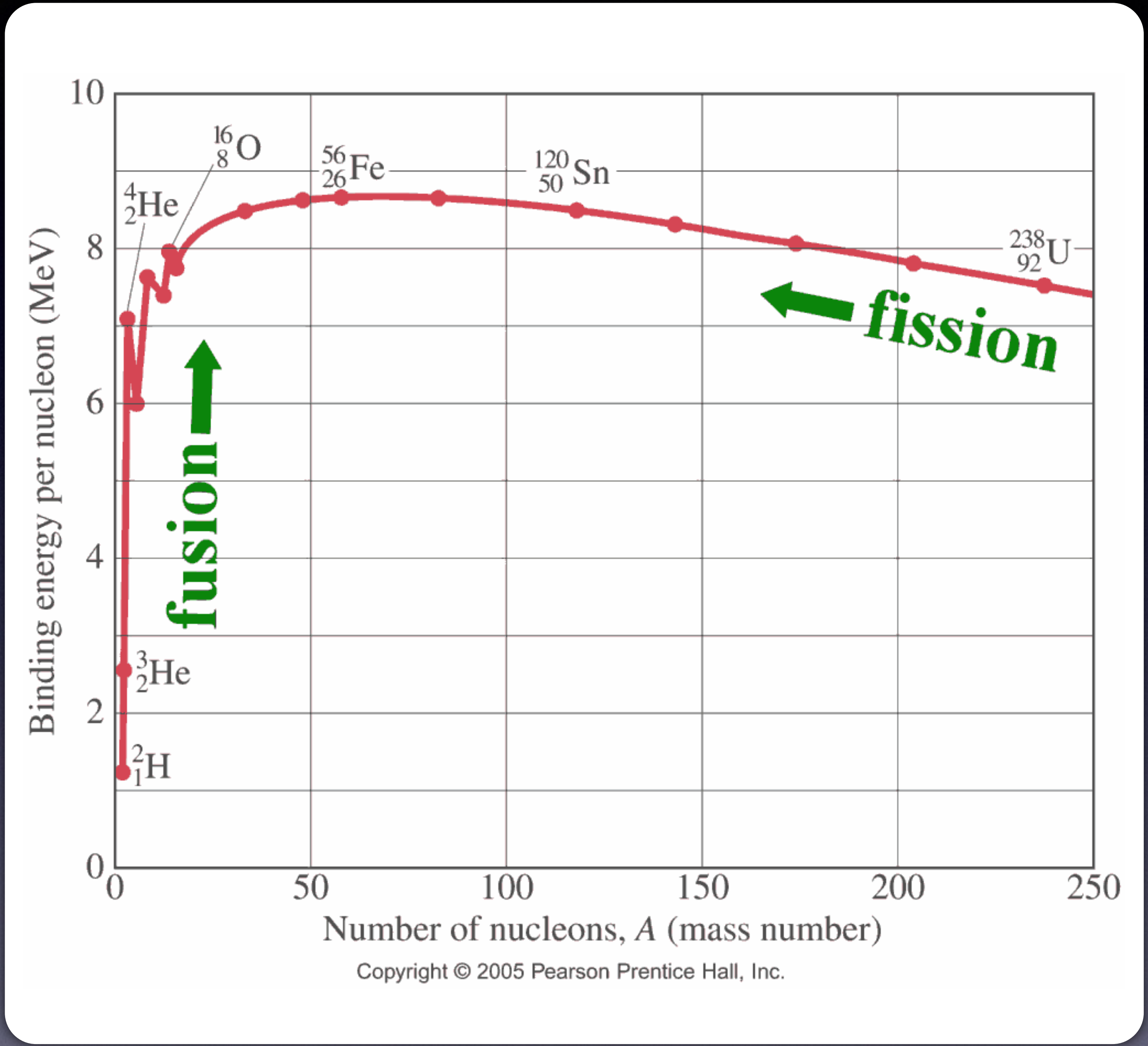
-10



Lecture Note by Pols

Nuclear binding energy

$$Eb = [Nm_N + Zm_p - m_i]\, c^2 > 0$$



Larger binding energy
= more stable

Fe has the largest
Eb/nucleon

Then, all the stars produce Fe? => No
Stellar material does not always behave as ideal gas

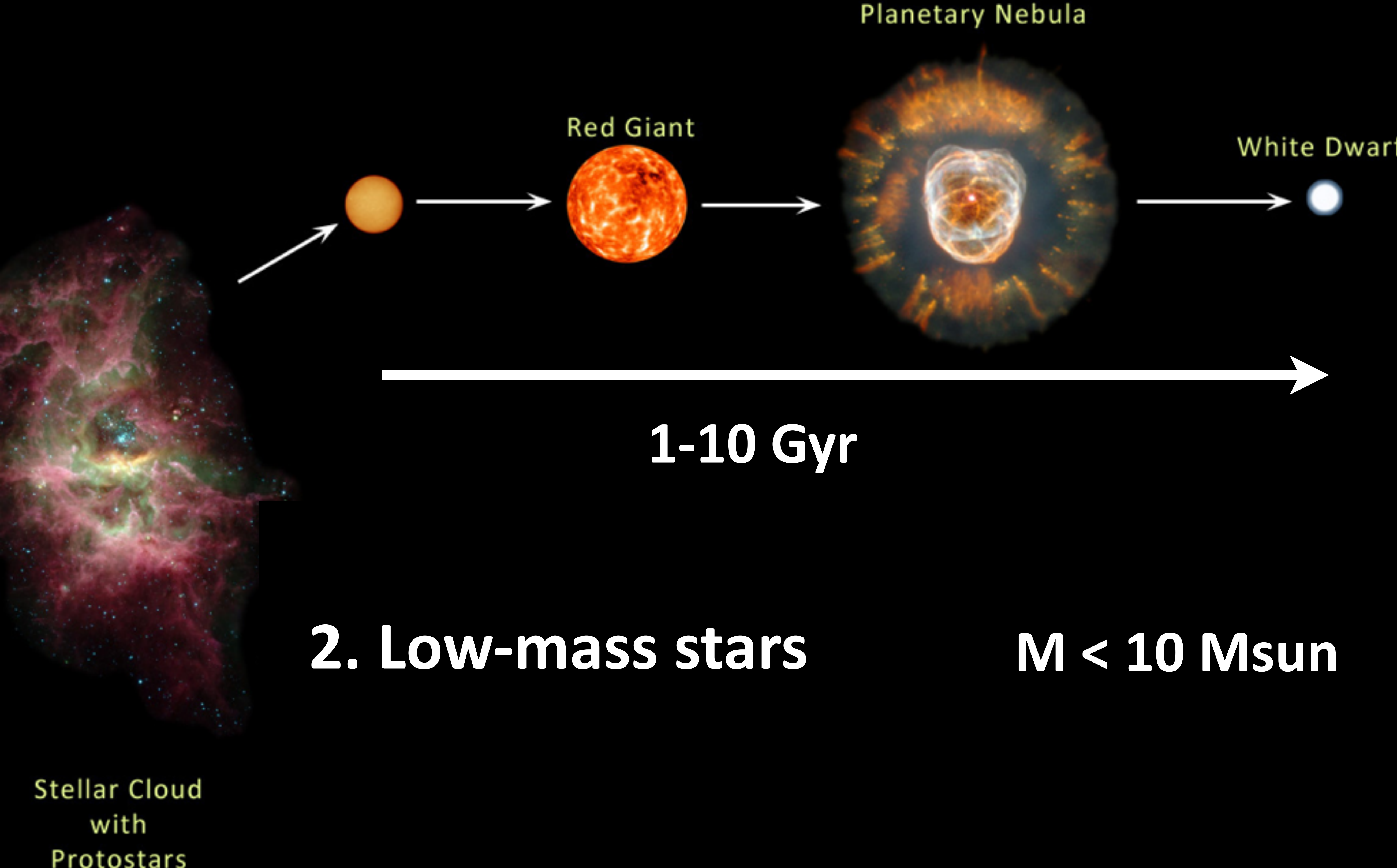
Phase 燃焼段階	Main reactions おもな反応	Products おもな生成物	T 温度（10^8 K）
H	pp チェイン CNO サイクル	^{4}He ^{14}N	0.15-0.2
He	$3\,^{4}He \longrightarrow {}^{12}C$ $^{12}C + {}^{4}He \longrightarrow {}^{16}O + \gamma$	^{12}C ^{16}O	1.5
C	$^{12}C + {}^{12}C \longrightarrow \begin{cases} ^{23}Na + p \\ ^{20}Ne + \alpha \end{cases}$	Ne,Na Mg,Al	7
Ne	$^{20}Ne + \gamma \longrightarrow {}^{16}O + \alpha$ $^{20}Ne + \alpha \longrightarrow {}^{24}Mg + \gamma$	O Mg	15
O	$^{16}O + {}^{16}O \longrightarrow \begin{cases} ^{28}Si + \alpha \\ ^{31}P + p \end{cases}$	Si,P,S, Cl,Ar,Ca	30
Si	$^{28}Si + \gamma \longrightarrow {}^{24}Mg + \alpha$ $^{24}Mg + \gamma \longrightarrow \begin{cases} ^{23}Na + p \\ ^{20}Ne + \alpha \end{cases}$ 多くの反応 ⟶ 統計平衡	Cr,Mn, Fe,Co, Ni,Cu	40

Nuclear statistical equilibrium

元素はいかにつくられたか（岩波書店）

Do all the stars evolve to Fe core?? => No

Equation of state is important
Stellar interior is not always ideal gas state



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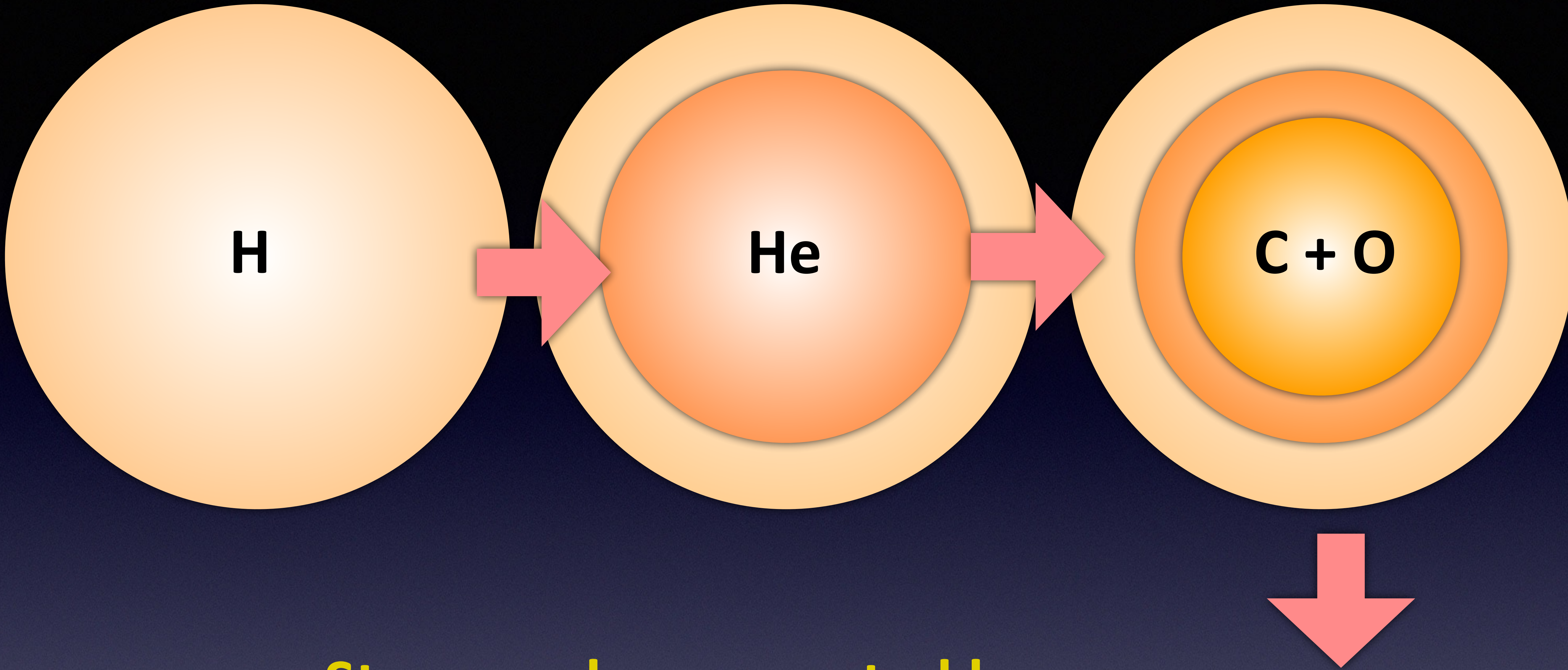
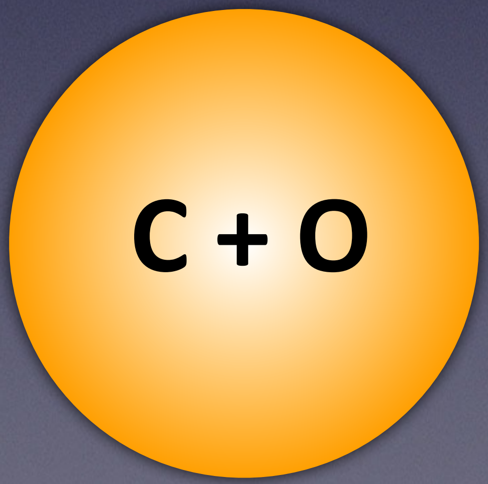


Stars can be supported by electron degeneracy pressure

White dwarf



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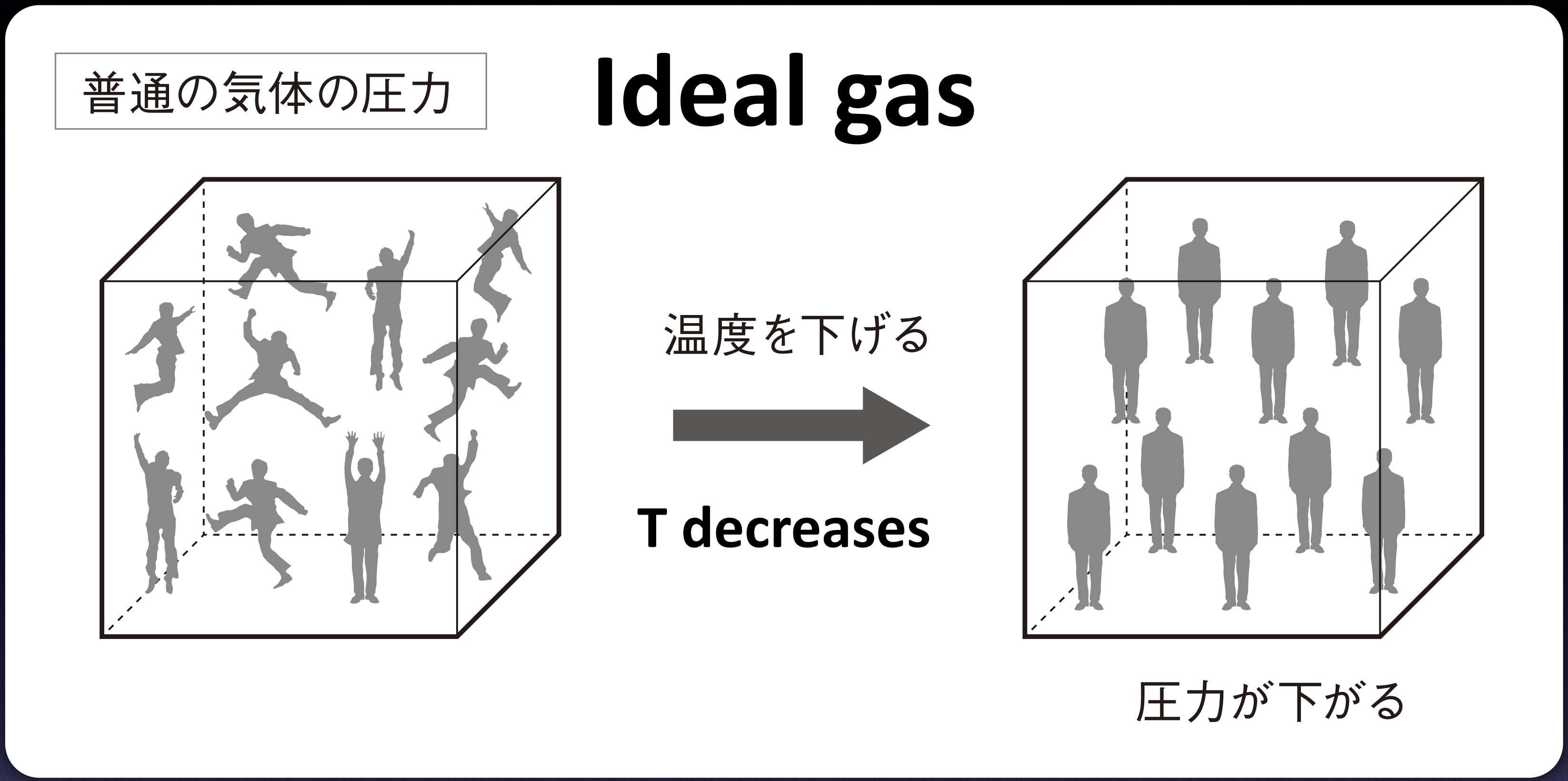
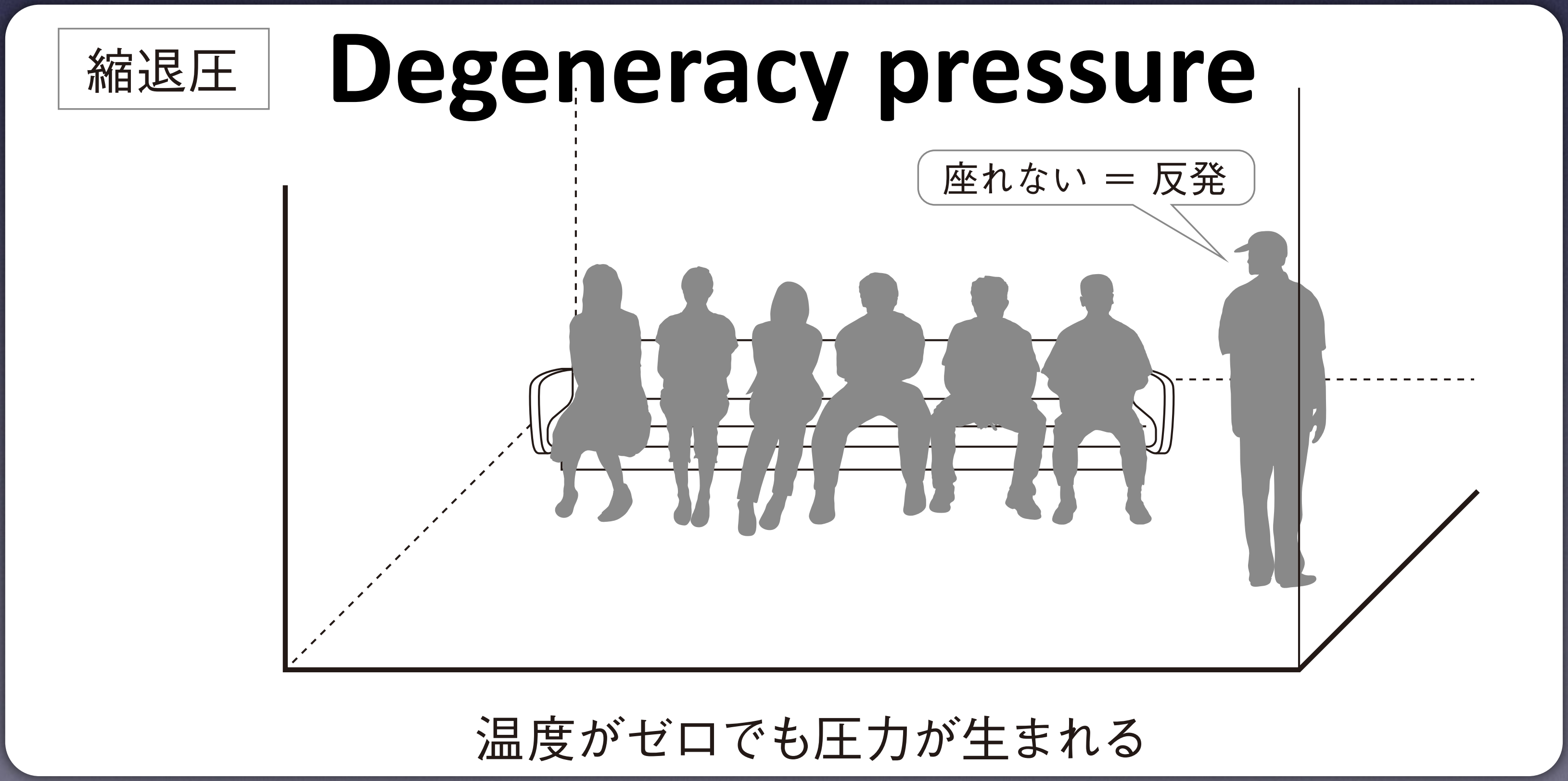
White dwarf: supported degeneracy pressure

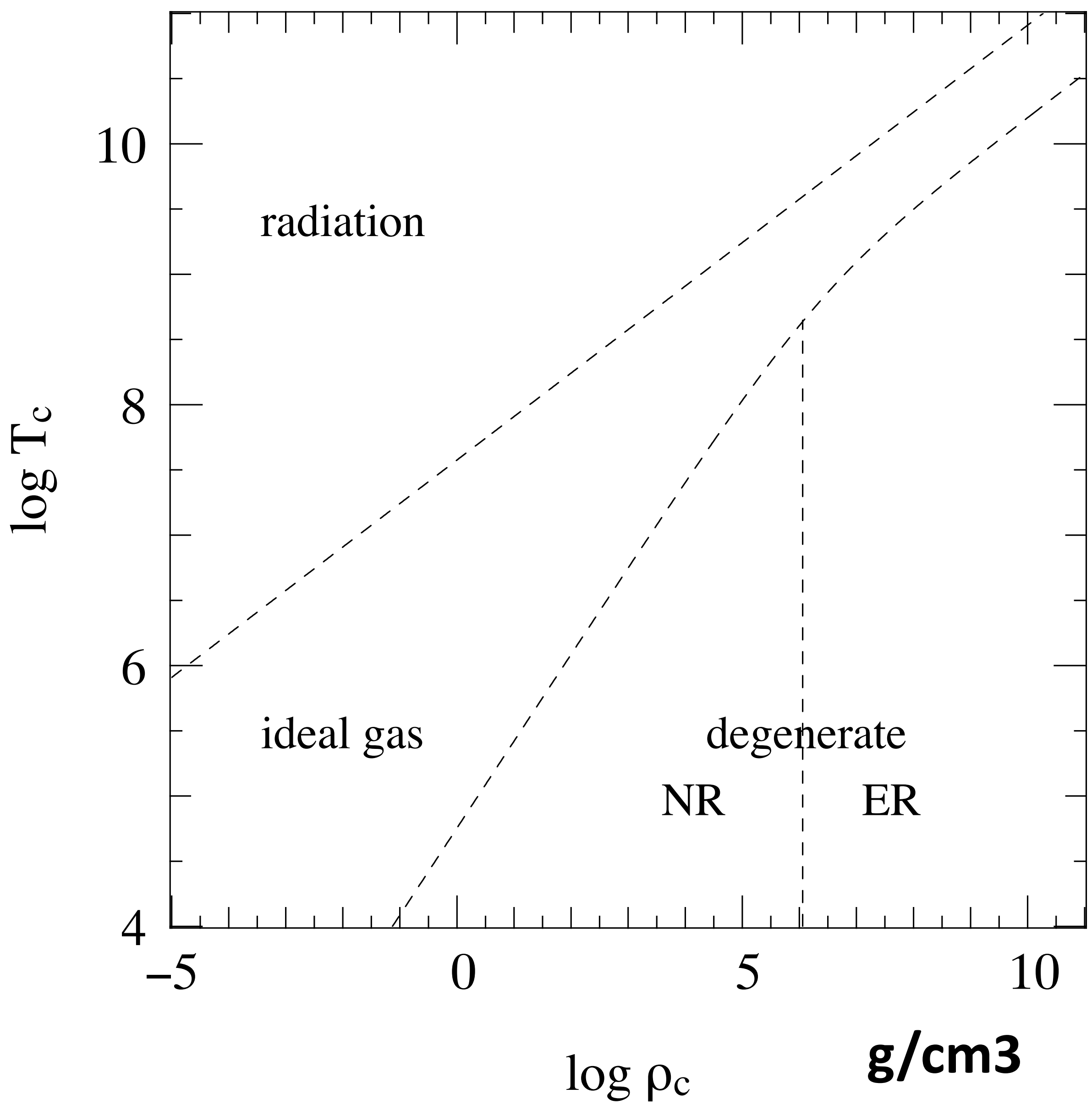




P is non-zero even at T=0

星が「死ぬ」とはどういうことか
（ベレ出版）

EOS



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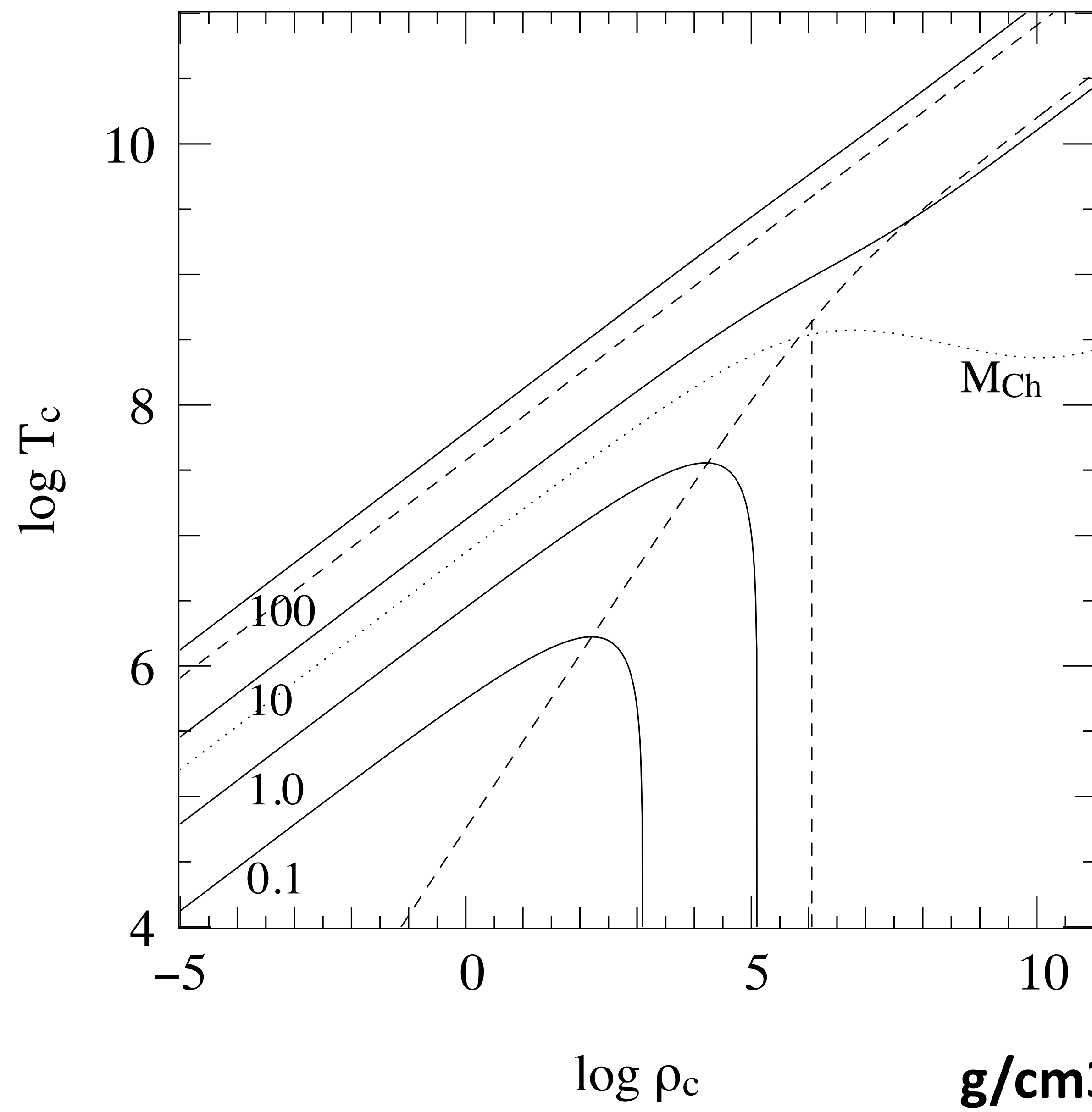
Evolution of T_c and ρ_c



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textbook by Pols

Assignment 2

For those who have not taken stellar evolution in undergrad course

2a. Derive pressure of ideal gas from the Maxwell distribution

2b. Derive pressure of degenerate electrons
(both for non-relativistic case and relativistic case)

2c. Derive radiation pressure from Planck function

2d. Draw the regions where

- ideal gas pressure
- degenerate pressure of non-relativistic electrons
- degenerate pressure of relativistic electrons
- radiation pressure

become dominant in the rho-T diagram.

レポート課題 2

学部の恒星物理学IIをとっていない人

2a. マクスウェル分布から
理想気体の圧力の式を導け

2b. 電子が非相対論的、超相対論的なときの
縮退圧の式を導き、実際に数字を入れて計算せよ

2c. プランク関数から輻射圧の式を導け

2d. 密度 - 温度平面で

- 理想気体のガス圧
- 電子の縮退圧（非相対論的）
- 電子の縮退圧（超相対論的）
- 輻射圧

がそれぞれ支配的になる境界を求め、図示せよ

Assignment 2

For those who have taken stellar evolution in undergrad course

Please attend some part of the conference

“ELT Science in Light of JWST” at Katahira from June 3-7.

Summarize the one of the invited talks you got interested in
(e.g., specification of TMT/GMT/ELT, some science cases)

about 2 pages, A4

レポート課題 2

学部の恒星物理学IIをとった人

6月3-7日に片平キャンパスで行われる
“ELT Science in Light of JWST” (の一部)に参加して
興味のある招待講演の内容をまとめる
(e.g., TMT/GMT/ELTのスペック, サイエンスケース)

A4で2ページ程度

Summary: Stellar evolution

- **Virial theorem (for ideal gas case)**
  - Internal energy always relates with gravitational energy
  - When stars lose energy, they contract
  - Temperature rises ("negative heat capacity")
- **Evolution of density and temperature**
  - Rise in temperature due to contraction $T \sim \rho^{1/3}$
  - Next burning stages => Onion-like structure
- **Importance of the equation of state**
  - Stars stop contraction if supported by degeneracy pressure => No temperature rise => End of nuclear burning
  - The core of low mass stars become a white dwarf

Appendix

1a. H-burning (pp chain)

$$4\,^{1}\mathrm{H} \rightarrow \,^{4}\mathrm{He} + 2\,e^{+} + 2\,\nu$$

$$^{1}\mathrm{H} + \,^{1}\mathrm{H} \rightarrow \,^{2}\mathrm{H} + e^{+} + \nu$$

$$^{2}\mathrm{H} + \,^{1}\mathrm{H} \rightarrow \,^{3}\mathrm{He} + \gamma$$

$$^{3}\mathrm{He} + \,^{3}\mathrm{He} \rightarrow \,^{4}\mathrm{He} + 2\,^{1}\mathrm{H}$$

pp1

$$^{3}\mathrm{He} + \,^{4}\mathrm{He} \rightarrow \,^{7}\mathrm{Be} + \gamma$$

$$^{7}\mathrm{Be} + e^{-} \rightarrow \,^{7}\mathrm{Li} + \nu$$

$$^{7}\mathrm{Li} + \,^{1}\mathrm{H} \rightarrow \,^{4}\mathrm{He} + \,^{4}\mathrm{He}$$

pp2

$$^{7}\mathrm{Be} + \,^{1}\mathrm{H} \rightarrow \,^{8}\mathrm{B} + \gamma$$

$$^{8}\mathrm{B} \rightarrow \,^{8}\mathrm{Be} + e^{+} + \nu$$

$$^{8}\mathrm{Be} \rightarrow \,^{4}\mathrm{He} + \,^{4}\mathrm{He}$$

pp3

Textbook by Pols

Energy production rate (per gram)

$q \sim \rho T^4$

$T \sim 4 \times 10^6$ K

$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu$

$^2H + {}^1H \rightarrow {}^3He + \gamma$

$^3He + {}^3He \rightarrow {}^4He + 2\,{}^1H$

pp1

$^3He + {}^4He \rightarrow {}^7Be + \gamma$

$^7Be + e^- \rightarrow {}^7Li + \nu$

$^7Li + {}^1H \rightarrow {}^4He + {}^4He$

pp2

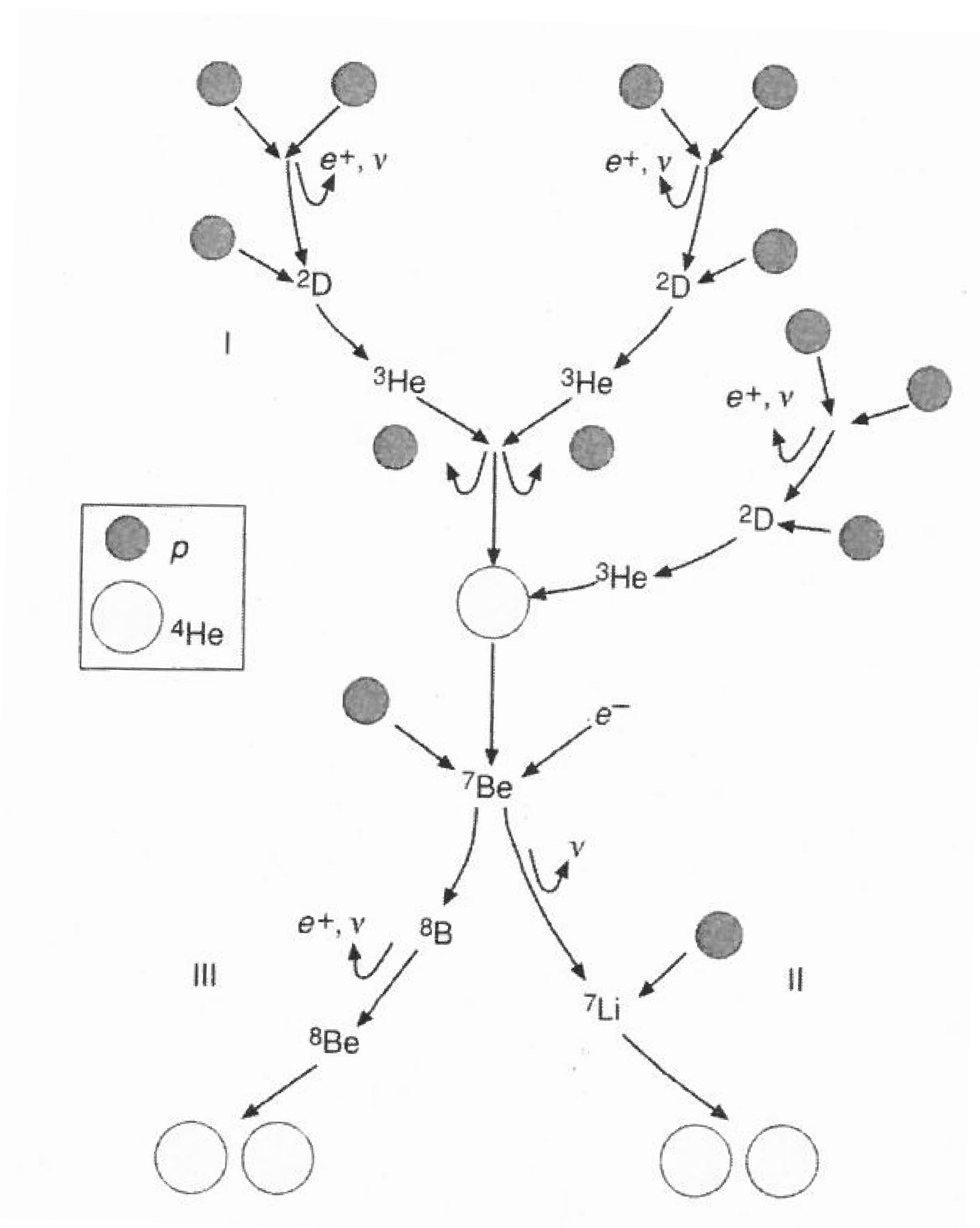
$^7Be + {}^1H \rightarrow {}^8B + \gamma$

$^8B \rightarrow {}^8Be + e^+ + \nu$

$^8Be \rightarrow {}^4He + {}^4He$

pp3

Textbook by Pols



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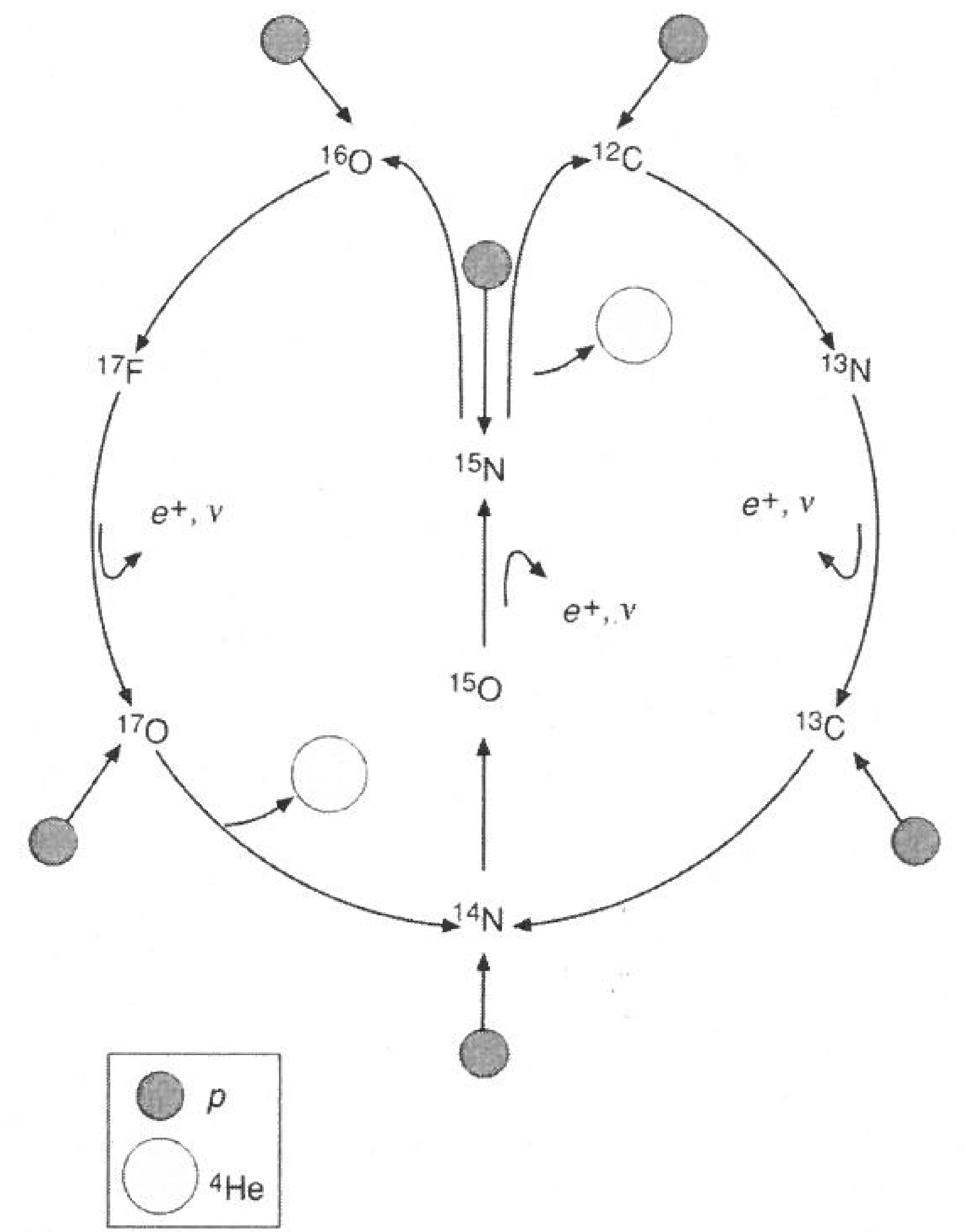
1b. H burning (CNO cycle)

E production rate $q \sim \rho T^{16}$

$T \sim 1.5 \times 10^{7}$ K

$$^{12}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{13}\mathrm{N} + \gamma$$
$$^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + e^{+} + \nu$$
$$^{13}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{14}\mathrm{N} + \gamma$$
$$^{14}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{15}\mathrm{O} + \gamma$$
$$^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + e^{+} + \nu$$
$$^{15}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}$$
$$\rightarrow {}^{16}\mathrm{O} + \gamma$$
$$^{16}\mathrm{O} + {}^{1}\mathrm{H} \rightarrow {}^{17}\mathrm{F} + \gamma$$
$$^{17}\mathrm{F} \rightarrow {}^{17}\mathrm{O} + e^{+} + \nu$$
$$^{17}\mathrm{O} + {}^{1}\mathrm{H} \rightarrow {}^{14}\mathrm{N} + {}^{4}\mathrm{He}$$

Textbook by Pols



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2. He-burning (triple alpha)

$$^{4}\mathrm{He} + {}^{4}\mathrm{He} \leftrightarrow {}^{8}\mathrm{Be}$$

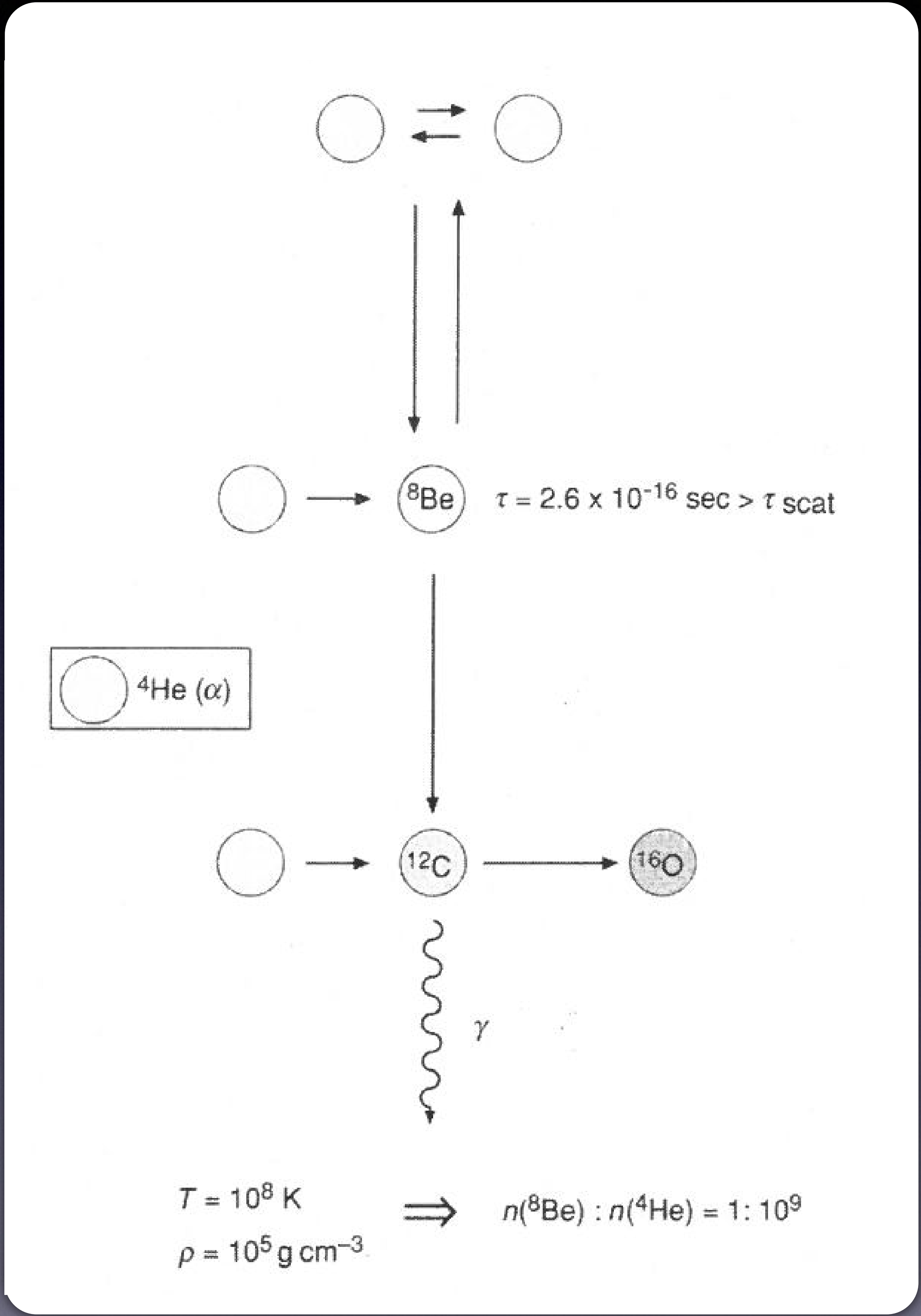
$$^{8}\mathrm{Be} + {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C}^{*} \rightarrow {}^{12}\mathrm{C} + \gamma$$

$$^{12}\mathrm{C} + {}^{4}\mathrm{He} \rightarrow {}^{16}\mathrm{O} + \gamma,$$

Energy production rate (per gram)

$q \sim \rho^2 T^{40}$

$T \sim 1.5 \times 10^8$ K



Textbook by Prialnik

3. C-burning

$$^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{24}\mathrm{Mg}^* \rightarrow {}^{20}\mathrm{Ne} + \alpha$$
$$\rightarrow {}^{23}\mathrm{Na} + \mathrm{p}$$

T ~ 7 x 10^8 K

4. Ne-burning

$$^{20}\mathrm{Ne} + \gamma \leftrightarrow {}^{16}\mathrm{O} + \alpha$$
$$^{20}\mathrm{Ne} + \alpha \rightarrow {}^{24}\mathrm{Mg} + \gamma$$

T ~ 1.5 x 10^9 K

5. O-burning

$$^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{32}\mathrm{S}^* \rightarrow {}^{28}\mathrm{Si} + \alpha$$
$$\rightarrow {}^{31}\mathrm{P} + \mathrm{p}$$

T ~ 2-3 x 10^9 K

6. Si-burning (Nuclear statistical equilibrium)

T > 4 x 10^9 K

High temperature => photo-dissociation

$$^{28}\mathrm{Si}\,(\gamma,\alpha)\,^{24}\mathrm{Mg}\,(\gamma,\alpha)\,^{20}\mathrm{Ne}\,(\gamma,\alpha)\,^{16}\mathrm{O}\,(\gamma,\alpha)\,^{12}\mathrm{C}\,(\gamma,\alpha)\,2\alpha$$

He capture

$$^{28}\mathrm{Si}\,(\alpha,\gamma)\,^{32}\mathrm{S}\,(\alpha,\gamma)\,^{36}\mathrm{Ar}\,(\alpha,\gamma)\,^{40}\mathrm{Ca}\,(\alpha,\gamma)\,^{44}\mathrm{Ti}\,(\alpha,\gamma)\,\ldots\,^{56}\mathrm{Ni}$$

=> equilibrium of many reactions

(Ex.)

$$^{28}\mathrm{Si} + \gamma \leftrightarrow {}^{24}\mathrm{Mg} + \alpha,$$

Nuclei with high binding energy tend to be produced (Fe, Co, Ni)